Cambridge International Examinations
Cambridge Pre-U Certificate

MATHEMATICS **9794/01**

Paper 1 Pure Mathematics 1 **May/June 2017**

MARK SCHEME

Maximum Mark: 80

Published

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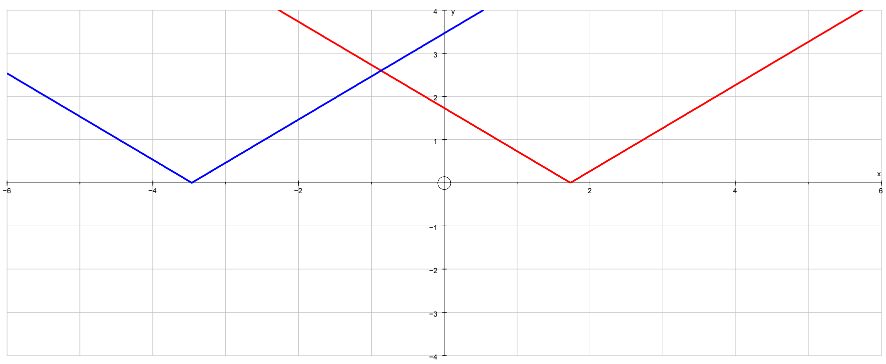
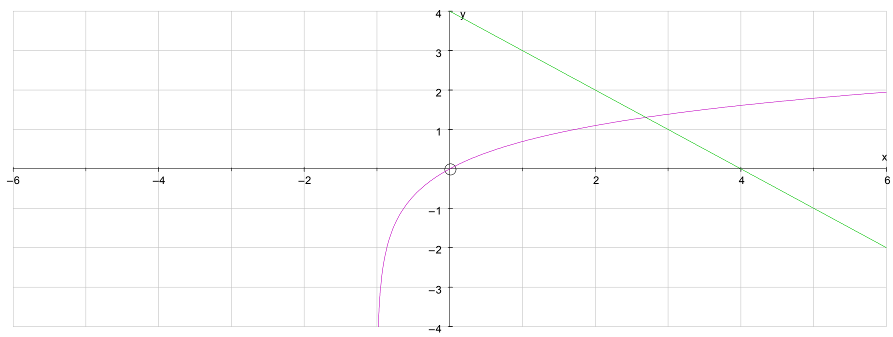
Question	Answer	Marks
1(i)	State (3, 2)	**B1**
1(ii)	Substitute (0, 2) to find diameter.	**M1**
	Obtain 6.	**A1**
2(i)	Apply correctly at least one logarithm law.	**M1**
	Obtain $\log 6$	**A1**
2(ii)	Apply the power law correctly at least once	**M1***
	Correctly combine log terms	**depM1**
	Obtain $\log\left(\frac{x^2z^2}{y^3}\right)$	**A1**
3(i)	Use $a^2 = b^2 + c^2 - 2bc\cos A$ but data may be in wrong position	**M1**
	Obtain $8^2 = 7^2 + 6^2 - 2(7)(6)\cos ABC$ or equivalent	**A1**
	Derive correctly $\cos ABC = 0.25$ AG	**A1**
3(ii)	State $\frac{1}{2}ab\sin C$ for the area of a triangle	**M1**
	Obtain correctly $\sin ABC$ (may be via angle ABC (= 75.5°) or an identity)	**M1**
	Obtain answers rounding to 20.3 (cm^2)	**A1**
	Alternative Use cosine rule to find another angle (angle A = 46.567, angle C = 57.91)	**M1**
	Find height of triangle (5.083)	**M1**
	Use 0.5(base)(height) = 20.3	**A1**
4	Use of the identity $\sin 2x = 2\sin x\cos x$	**B1**
	Obtain $\sin x = \frac{\sqrt{3}}{2}$	**B1**
	Obtain 60° and 120°	**B1**
	Obtain 90° and 270°	**B1**

Question	Answer	Marks
5	Attempt to square and expand brackets with 3 terms resulting from each.	M1
	Obtain $x^2 - 2\sqrt{3}\,x + 3$	A1
	Obtain $x^2 + 4\sqrt{3}\,x + 12$	A1
	Rearrange to make x the subject.	M1
	$x > \dfrac{-\sqrt{3}}{2}$ aef.	A1
	Alternative 1 : an approach based on a piecewise function Consider at least two intervals $-(x-\sqrt{3}) - -(x+2\sqrt{3}) < 0$ $-(x-\sqrt{3}) - (x+2\sqrt{3}) < 0$ $(x-\sqrt{3}) - (x+2\sqrt{3}) > 0$	M1
	Specify the intervals $(-\infty, -2\sqrt{3}), (-2\sqrt{3}, \sqrt{3}), (\sqrt{3}, \infty)$	A1
	Discard the first and last intervals, may be without comment	M1
	Solve $-(x-\sqrt{3}) - (x+2\sqrt{3}) < 0$ or equiv	M1
	$x > \dfrac{-\sqrt{3}}{2}$ with no incorrect working. Extra intervals M1M0M1 only	A1
	Alternative 2 : an approach based on graphs only 	
	$y = \lvert x - \sqrt{3}\rvert$ drawn with intersections with axes shown	M1A1
	$y = \lvert x + 2\sqrt{3}\rvert$ drawn with intersections with axes shown	M1A1
	$x > \dfrac{-\sqrt{3}}{2}$	A1

Question	Answer	Marks
6(i)	Attempt $1+\frac{1}{2}x+\frac{\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)}{2}x^2+\frac{\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)}{6}x^3$ (allow omission of brackets at this stage) but must reach the x^3 term	M1
	Obtain $1+\frac{1}{2}x$	A1
	Obtain $\frac{-1}{8}x^2$	A1
	Obtain $\frac{1}{16}x^3$	A1
6(ii)	Attempt sum of two relevant terms Must see the sum of two terms only each giving an x^3 result	M1
	Obtain $\frac{1}{8}-\frac{k}{8}=1$	A1
	Obtain $k=-7$	A1
7(i)	State **translation** – NOT "shift" or "move"	B1
	… one unit to the left OR in the negative direction OR to the left by 1 OR 1 unit parallel to the x axis OR by specifying the vector $\begin{pmatrix}-1\\0\end{pmatrix}$.	B1
	Stating "in" OR "on " OR "along" the x-axis" OR "a factor of –1" B0	
7(ii)	Sketch ln graph with asymptote at $x=-1$ clearly indicated.	B1
	Sketch correct $y=4-x$ to show intersection points with the axes clearly.	B1
		
	State one intersection implies one root.	B1*

Question	Answer	Marks
7(iii)	State derivative is $\frac{1}{1+x}+1$	B1
	Use $x_{n+1} = x_n - \dfrac{\ln(1+x_n)-4+x_n}{\dfrac{1}{1+x_n}+1}$	M1
	Obtain at least $x_1 = 2.676$	A1
	State 2.693 explicitly	A1
	Alternative Using function $f(x) = e^{4-x} - x - 1$. Derivative $= -e^{4-x} - 1$ with $x_1 = 2.523$ then 2.693	4
8	Obtain $3y^2\frac{dy}{dx} + 12y\frac{dy}{dx}$	B1
	Obtain $-2\frac{dy}{dx} = 6x+2$	B1
	Substitute (1, 1) into their $\frac{dy}{dx}$ as long as valid implicit differentiation used	M1
	Use $m_1m_2 = -1$	M1
	Obtain $\frac{-13}{8}$	A1
	Use $(y-1) = m(x-1)$	M1
	Obtain $8y + 13x - 21 = 0$	A1
	Unclear notation used or apparent slips in working but otherwise correct. Award final A0	

Question	Answer	Marks
9	State $z = 2$ as a root either from the factor theorem or in a list of all 3 roots, No working required. (8 + 12 – 20 = 0 seen with no indication of $z = 2$ as a root B0 " $z - 2$ is a root" or "$z = 2$ is a factor" B0 even if $z = 2$ listed as a root later. If factors only ever seen B0 and later A0 also.	**B1**
	Attempt long division to obtain a quadratic factor. Substituting $z = a + \mathrm{i}b$ must end with correct expressions, e.g. $-8a^3 - 12a - 20 = 0$ and $2a^3 + 3a + 5 = 0$	**M1**
	Obtain $z^2 + 2z + 10$	**A1**
	Use quadratic formula to solve their quadratic	**M1**
	Obtain $-1 + 3\mathrm{i}$ and $-1 - 3\mathrm{i}$	**A1**
	State $-1 + 3\mathrm{i}$ has modulus $\sqrt{10}$ and argument 1.89 or 108° [Allow arguments between 0 and 2π] Do not accept arguments given in final form as $\tan^{-1}(-3)$ or $\tan^{-1}(+3)$	**B1**
	State $-1 - 3\mathrm{i}$ has modulus $\sqrt{10}$ and argument −1.89 or 4.39 or −108° or 252°	**B1**
	State 2 has modulus 2 and argument 0	**B1**
	Three correct points shown on an Argand diagram. Do not accept a plainly Cartesian graph. If no labels, then must indicate the points as complex numbers, even as z_1 z_2 as long as clear from a list of roots. Accept a cross or similar for 2	**B1**
10	Rearrange to obtain $x^2 = 9 - 3y$ and $x^2 = 9 - 5y$	**B1**
	Use *their* (π) $\int x^2 (\mathrm{d}y)$ on separate integrals	**M1**
	Obtain $9y - \frac{3}{2}y^2$ and $9y - \frac{5}{2}y^2$	**A1**
	Use limits (3, 0) and (1.8, 0) on separate integrals in correct order	**M1**
	Obtain 13.5π and 8.1π	**A1**
	Subtract separate volumes in correct order	**M1**
	Obtain $\frac{27\pi}{5}$ or equiv (16.96 or 5.4π)	**A1**

Question	Answer	Marks
10	**Alternative method** Form a single integral by subtraction $y=\frac{1}{3}(9-x^2)-\frac{1}{5}(9-x^2)=\frac{2}{15}(9-x^2)$	**M1A1**
	Rearrange to x^2 form ($x^2=9-\frac{15y}{2}$)	**M1**
	Use (π) $\int x^2\mathrm{d}y$	**M1**
	Obtain $9y-\frac{15}{4}y^2$	**A1**
	Use limits (1.2, 0) on a single integral in correct order	**M1**
	Obtain $\frac{27\pi}{5}$	**A1**
	Special Ruling	
	Rotation about the x-axis : State $\pi\int_0^3\frac{1}{9}(9-x^2)^2\,\mathrm{d}x-\pi\int_0^3\frac{1}{25}(9-x^2)^2\,\mathrm{d}x$ = B1 and final answer 28.95 B2	
11(i)	State $\overrightarrow{OQ}$ = 6**i** – 4**j** – 2**k** and $\overrightarrow{OP}$ = 6**i** + 3**j** – 9**k** or AQ = 4**i** − 5**j** + **k** and BP = 3**i** + 5**j** − 8**k.**	**B1**
	Form equation of line AQ and BP in form **a** + λ**b**	**M1**
	Obtain $\mathbf{r}_{AQ}$ = (2**i** + **j** – 3**k**) + λ(4**i** – 5**j** + **k**) Could use OQ so $\mathbf{r}_{OQ}$ = (6**i** − 4 **j** – 2**k**) + λ(4**i** – 5**j** + **k**) giving $\mu=\frac{1}{5}$ or $\lambda=\frac{-3}{5}$ OR OQ and OP = (6**i** + 3**j** – 9**k**) + λ(3**i** + 5**j** − 8**k**) giving $\mu=\frac{-4}{5}$ or $\lambda=\frac{-3}{5}$ OR AQ and BP (2**i** + **j** – 3**k**) + λ(4**i** – 5**j** + **k**) and (6**i** + 3**j** – 9**k**) + λ(3**i** + 5**j** − 8**k**) giving $\mu=\frac{-4}{5}$ or $\lambda=\frac{2}{5}$	**A1**
	Obtain $\mathbf{r}_{BP}$ = (3**i** − 2**j** − **k**) + μ(3**i** + 5**j** – 8**k**)	**A1**
	Equate line equations and solve two eqns simultaneously to find some value of λ or μ	**M1**
	Obtain either $\mu=\frac{1}{5}$ or $\lambda=\frac{2}{5}$	**A1**
	State $\left(\frac{18}{5},-1,\frac{-13}{5}\right)$ Must be in coordinate form	**B1**

Question	Answer	Marks
11(i)	**ALTERNATIVE**	
	$\overrightarrow{OQ} = 6\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$ and $\overrightarrow{OP} = 6\mathbf{i} + 3\mathbf{j} - 9\mathbf{k}$	**B1**
	AQ and **BP** intersect at M Then **OM = OA + AM = OB + BM**	**M1**
	$= \mathbf{a} + \lambda\mathbf{AQ} = \mathbf{a} + \lambda(-\mathbf{a}+2\mathbf{b})$	**A1**
	$= \mathbf{b} + \mu\mathbf{BP} = \mathbf{b} + \mu(-\mathbf{b} + 3\mathbf{a})$	**A1**
	$(1 - \lambda)\mathbf{a} + 2\lambda\mathbf{b} = (1 - \mu)\mathbf{b} + 3\mu\mathbf{a}$	**M1**
	Obtain either $\mu = \frac{1}{5}$ or $\lambda = \frac{2}{5}$	**A1**
	State $\left(\frac{18}{5}, -1, \frac{-13}{5}\right)$ Must be in coordinate form	**B1**
11(ii)	Use dot product correctly to find an angle	**M1**
	Obtain either $\lvert\overrightarrow{AQ}\rvert = \sqrt{42}$ or $\lvert\overrightarrow{BP}\rvert = \sqrt{98}$	**B1**
	Obtain 70.9°	**A1**
12	State $P = \frac{\pm k}{V}$	**B1**
	Find k by substituting $P = 5$ and $V = 80$	**M1**
	$P = \frac{400}{V}$ May be implied by correct working or $k = 400$	**A1**
	Differentiate a correct expression for P: $\frac{dP}{dV} = \frac{-400}{V^2}$	**M1**
	State $\frac{dV}{dt} = 10$ or implied by use in $\frac{dP}{dt} = \frac{dP}{dV} \times \frac{dV}{dt}$	**B1**
	Use $\frac{dP}{dt} = \frac{dP}{dV} \times \frac{dV}{dt}$ to obtain an expression in V (OR $\frac{dV}{dP} = \frac{dV}{dt} \times \frac{dt}{dP}$ giving $\frac{-400}{P^2} = 10 \times \frac{dt}{dP}$ to obtain an expression in P M1 and substitute $P = 5$ for M1)	**M1**
	Substitute $V = 80$ into correct $\frac{dP}{dV} = \frac{-400}{V^2}$	**M1**
	Obtain 0.625 (pascals)	**A1**

Question	Answer	Marks
12	**Alternative** State $P = \dfrac{\pm k}{V}$	B1
	Find k by substituting $P = 5$ and $V = 80$	M1
	$P = \dfrac{400}{V}$ May be implied by correct working or $k = 400$	A1
	$V = \int 10\,\mathrm{d}t \Rightarrow V = 10t + c$ At $t = 0, V = 80$ so $V = 10t + 80$	B1
	$\dfrac{\mathrm{d}P}{\mathrm{d}t} = \dfrac{-400 \times 10}{(10t + 80)^2}$ or $\dfrac{\mathrm{d}P}{\mathrm{d}t} = \dfrac{-40}{(t + 8)^2}$	M1M1
	At $t = 0$ $\dfrac{\mathrm{d}P}{\mathrm{d}t} = -0.625$	A1
	Final answer 0.625	A1